NOTATION

x, y, Cartesian coordinates; U, roller peripheral velocity; W, matrix translational velocity; h, current gap height; L, characteristic length of flow zone; v_x, v_y, velocity components; K, matrix permeability coefficient; P, $\bar{P}$, dimensional and dimensionless pressures; μ, n, rheological constants; τ_{xy}, shear stress; R, roller radius; x_0, x_1, mixing zone characteristic points; ρ, dimensionless Gaskell variable; ρ_0, λ, coordinates of flow zone boundary; Γ, dimensionless matrix permeability; Y, dimensionless ordinate; Ψ, $\bar{\Psi}$, dimensional and dimensionless flow functions; f, friction; A, a function of ρ; m, number of point; $I(\rho)$, integral function; $\Delta\rho$, step in ρ; Q, volume granulate output; F, tensile force; N, required power; C, a function of ρ; C_*, value of function C at stagnation point; ρ_*, dimensionless coordinate of pressure maximum; c_1, a function of x; α, central angle; $\bar{Q}$, $\bar{F}$, $\bar{N}$, dimensionless output, tensile force, and required power; b, working width of roller.

LITERATURE CITED

1. N. P. Sigaev, "Study of viscous mass flows for calculation of roller type granulators," Candidate's Dissertation, Moscow (1976).
2. S. G. Aleksandrova and O. N. Ivanov, Machines and Technology for Processing Rubber, Polymer, and Resin Mixtures [in Russian], Mezhvuz. Sb. Yaroslavl' (1984), pp. 8-11.
3. T. A. Partalin and O. N. Ivanov, Design, Study, and Calculation of Chemical Production Equipment [in Russian], Mezhvuz. Sb. Moskvy (1980), pp. 121-125.
4. É. Bernkhardt, Processing of Thermoplastic Materials [in Russian], Moscow (1965).
5. D. M. MacKelvy, Polymer Processing [Russian translation], Moscow (1965).

APPROXIMATION OF THE GENERALIZED BUCKINGHAM EQUATION

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The generalized Buckingham equation is approximated by a quadratic function for media describable by the Balkley–Gershel model.

The Balkley–Gershel equation of state [1] is a quite general rheological law which describes the behavior of various high concentration suspensions:

$$\tau = \tau_0 \operatorname{sign} \frac{du}{dr} + k \left| \frac{du}{dr} \right|^{n-1} \frac{du}{dr}.$$

For a laminar flow regime in a circular tube the relationship between volume flow rate and friction on the wall is defined by the generalized Buckingham equation [2]

$$Q = \pi R^3 \frac{n}{3n+1} \left(\frac{\tau_w}{k} \right)^{1/n} (1-\bar{r}_p)^{\frac{n+1}{n}} \left[1 + \frac{2n}{2n+1} \bar{r}_p + \frac{2n^2}{(n+1)(2n+1)} \bar{r}_p^2 \right]. \quad (1)$$

Use of Eq. (1) in practical calculations is difficult, since it is usually necessary to define the pressure drop ΔP in terms of the volume flow rate, i.e., $\Delta P = f(Q)$.

We will approximate the auxiliary function $\varphi_1(x)$ by the quadratic expression $\varphi_2(x)$:

$$\varphi_1(x) = (1-x)^{n+1} \left(1 + \frac{2n}{2n+1} x + \frac{2n^2}{(n+1)(2n+1)} x^2 \right)^n, \quad (2)$$

$$\varphi_2(x) = (ax^2 + bx + c)^{1/2} + d + ex. \quad (3)$$

It is evident that the expression $\varphi_1^{1/n}(x)$ coincides with the terms in square brackets in Eq. (1).

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We define the coefficients a, b, c, d, and e from the conditions of equality of values of functions and their derivatives on the boundaries of the interval [0, 1]:

$$\varphi_1(0) = \varphi_2(0); \quad \varphi_1(1) = \varphi_2(1); \quad \varphi_1'(0) = \varphi_2'(0); \quad \varphi_1'(1) = \varphi_2'(1).$$

and the condition that the relative deviation of the approximate value from the precise $\bar{\delta} = |(\varphi_1 - \varphi_2)/\varphi_1|$, be minimal for all values of x in the interval between zero and unity.

Performing the necessary computations we obtain:

$$a = 1 - 2d + e^2; \; b = 2(de + 2d - 1); \; c = (1 - d)^2;$$

$$e = -\frac{n}{2n+1} - \frac{n+1}{2n+1}d.$$

The still unknown value of the coefficient d can be determined from the condition of minimum error.

For the range n = 0-2 the values of the coefficients a, b, c, d, e and the corresponding error of the approximation for x = 0.9 are presented in rows 2-7 of Table 1.

Figure 1 shows a comparison of the exact function $\varphi_1(x)$ and the approximating function $\varphi_2(x)$ for various values of the nonlinearity parameter n.

Row 8 of Table 1 presents the error in approximating the function φ_1 by the function [2]

$$\varphi_3(x) = (ax^2 + bx + c)^{1/2} + d, \quad (4) \qquad (4)$$

which coincides with $\varphi_2(x)$, if we take e = 0 in the latter. It is evident that in this case the error of the approximation is significantly higher, i.e., the proposed approximation $\varphi_2(x)$ is more precise.

By substituting the expression $\varphi_2(x)$ in place of the two terms in square brackets in Eq. (1), we can write the function $\Delta P = f(Q)$ in explicit form.

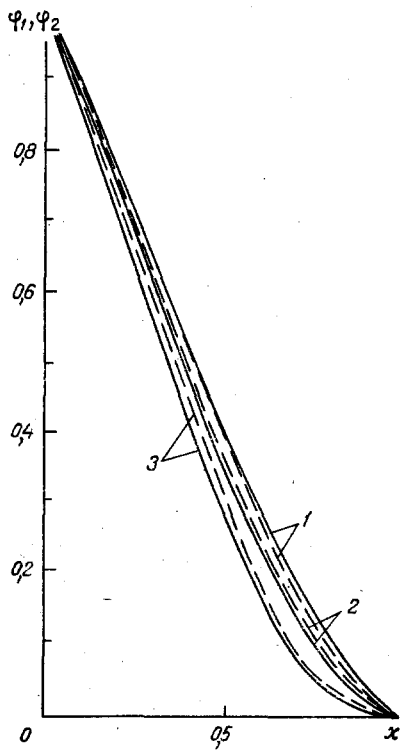

Fig. 1. Comparison of functions $\varphi_1(x)$ (solid curves) and $\varphi_2(x)$ (dashed) for n = 0.5 (1), 1 (2), 2 (3).

TABLE 1. Values of Coefficients of Approximating Function φ_2 and Errors in Approximation of φ_2, φ_3

n	0	0,1	0,2	0,5	0,75	1	1,5	2
a	0	0	899,69	384,67	35,32	1,457	0,691	0,597
b	0	0	—2086,01	—1004,36	—92,11	—2,759	—1,098	—0,8897
c	0	0	1211,04	661,00	62,41	1,464	0,4624	0,348
d	1	1	—33,8	—24,71	—6,90	—0,210	0,320	0,41
e	—1	—1	28,83	18,28	4,53	—0,193	—0,575	—0,646
$\bar{\delta}$, %	0	24,62	34,30	23,42	3,51	3,42	3,44	11,72
$\bar{\delta}$, %, $e=0$ [2]	0	37,0	45,69	39,40	21,80	6,57	96,09	273,03

If we introduce the notation $\Delta P = 2 L\tau_w/R$; $\Delta P_0 = 2 L\tau_0/R$, then the expression for the pressure drop over the pipe length L is given by

$$\Delta P = \Delta P_0 \frac{2((F-e)^2 - a)}{2(F-e)d + b + [(2(F-e)d + b)^2 - 4((F-e)^2 - a)(d^2 - c)]^{1/2}}, \tag{5}$$

where

$$F = \frac{k}{\tau_0}\left[\frac{(3n+1)Q}{\pi R^3 n}\right]^n. \tag{6}$$

In the special case n = 1 (Bingham liquid) Eq. (5) transforms to

$$\Delta P = \Delta P_0 \frac{\frac{1-2d}{32} I^2 - \frac{1+2d}{6} I - 2}{\frac{1-2d}{4} I - 2d - \left[4(1-d)^2 + \frac{(1-d)(1-2d)}{3} I\right]^{1/2}} \frac{8}{I}.$$

In the case of a power law liquid ($\Delta P_0 \to 0$) Eq. (5) transforms to the precise expression

$$\Delta P = \frac{2kL}{R}\left[\frac{u_\tau}{R}\left(\frac{3n+1}{n}\right)\right]^n.$$

The approximating function Eq. (5) obtained herein can be used to determine necessary pressure heads for pumping of media describable by the Balkley—Gershel model.

NOTATION

τ, shear stress; τ_w, shear stress on tube wall; τ_0, limiting shear stress; k, measure of liquid consistency; n, nonlinearity parameter; u, velocity; $u_m = Q/(\pi R^2)$, mean velocity; r, radius; R, tube radius; $r_p = \tau_0/\tau_w$, plug zone flow radius; Q, volume flow rate; ΔP, pressure drop over tube section of length L; ΔP_0, limiting pressure drop on tube section of length L; $\varphi_1(x)$, function defined by Eq. (2); $\varphi_2(x)$, function defined by Eq. (3); $\varphi_3(x)$, function defined by Eq. (4); x, variable; F, function defined by Eq. (6); $I = 2\tau_0 R/(\mu u_m)$, Il'yushin number for viscoplastic liquid.

LITERATURE CITED

1. Z. P. Shul'man, Convective Heat-Mass Transport of Rheologically Complex Liquids [in Russian], Moscow (1975).

2. V. E. Gubin and V. V. Gubin, Pipeline Transport of Petroleum and Petroleum Products [in Russian], Moscow (1982).

EXPERIMENTAL STUDY OF THE CHARACTERISTICS OF A MIXING D_2—CO_2-GDL

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Higher values of laser characteristics were obtained in a mixing D_2—CO_2-GDL than in N_2—CO_2-, N_2—N_2O-, and D_2—N_2O-mixing GDL with stagnation temperatures $T_0 \leq 1600$ K.

The D_2—CO_2 system is of interest for the development of low-temperature GDL with a high specific output energy. The distinguishing feature of this molecular couple is the existence of a large energy defect between the vibrational quantum of deuterium (Θ_4 = 4310 K) and the $00^0 1$ level of the CO_2 molecule (Θ_3 = 3380 K). The possibility of pumping the antisymmetric mode of CO_2 with deuterium through the combination level of CO_2 ($01^1 1$) was demonstrated in [1, 2]. The antisymmetric mode of CO_2 is heated up to temperatures exceeding the stagnation temperature of the gas flow as a result of such nonresonant exchange between the vibrational levels of the energy-carrying and radiating components. Gains of ~1 m^{-1} at $T_0 \approx 840$ K have been obtained in a GDL with a premixed mixture $D_2 + CO_2$ [2]. For $T_0 > 1000$ K, however, the chemical reaction between D_2 and CO_2, reducing the amount of CO_2 in the mixture and forming D_2O [4], can have a negative effect. For this reason it is of interest to study the scheme of a mixing GDL (MGDL) with selective heating of deuterium.

In [3, 4] the specific laser energy in D_2—CO_2 was found to be ~100 J/g for T_0 = 2000 K under the assumption of instantaneous mixing. The approximate character of the mixing model employed does not permit objective evaluation of the possibilities of this layer. Since the molecular masses of the mixing flows differ by a factor of 11 and deuterium is a strong deactivator of vibrational excitation in CO_2 [5], the mixing of D_2 and CO_2 can be accompanied by large losses of vibrational energy.

In this paper we present the results of an experimental study of an active medium consisting of D_2—CO_2 compared with N_2—CO_2-, N_2—N_2O-, and D_2—N_2O-media, obtained by gas-dynamic mixing, with stagnation pressures exceeding the atmospheric pressure.

The operation of a mixing GDL was modeled on a gas-dynamic stand with impulsive electric heating of the gas-energy carrier in a chamber with a finite volume (see Fig. 1). This type of heater was chosen because of the longer efflux time of the heated gas, for example, compared with shock tubes, which is significant in impulsive modeling of a mixing D_2—CO_2-GDL, since the large difference in the molecular weights can substantially increase the time for reaching the computed efflux regime in the nozzle.

The gas was heated by a pulsed electric discharge and, after a diaphragm was ruptured, flowed through the nozzle block of the mixing unit into the channel of the GDL (see Fig. 1). The stagnation temperature was calculated from the experimentally measured dependence of the pressure of the energy carrier in the heater taking into account the equations describing the consumption and state of the gas. The pressure P_0, the temperature T_0, and the consumption of the gas-energy carrier G decreased monotonically during the experiment. The corresponding parameters of the radiating gas with the diluent were maintained constant. The duration of the working pulse of the setup equaled 15-20 msec.

The effect of dissociation and recombination processes in the heater chamber accompanying the cooling of the gas on the magnitude of the vibrational energy stored in deuterium E_V was studied by a computational method. The vibrational excitation of the molecules was described

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